

Math 1320  
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Fall '14

NAME KEY

## Exam 1

Tuesday, September 23<sup>rd</sup>, 2014

#1	/20
#2	/20
#3	/20
#4	/20
#5	/20
Total	/100

You may use a calculator and a formula sheet on this exam. **Wherever necessary, you must show all of your work to receive full credit for a problem.**

1) A manufacturer sells computer chips for \$300 each. The company's fixed costs are \$150,000 per month, and marginal costs are \$180 per chip.

a) Write the equations for the revenue, cost, and profit functions. Let  $x$  be the number of chips.

$$R(x) = 300x$$

$$C(x) = 180x + 150000$$

$$\begin{aligned} P(x) &= 300x - (180x + 150000) \\ &= 120x - 150000 \end{aligned}$$

b) How many computer chips must be sold to break even? Round to the nearest chip.

$$120x - 150000 = 0$$

$$120x = 150000$$

$$x = 1250 \text{ chips}$$

2) A video rental store owner finds that he is able to rent 20 DVDs per day if the price is set at \$4 per DVD, and 50 DVDs per day if the price is \$1 per DVD. His supplier is willing to supply 50 DVDs per day if the price is \$5 per DVD, and 20 videos if the price is \$2 per DVD.

a) Write down the associated linear demand and supply functions.

Demand

$$q = mp + b$$

$$m = \frac{50 - 20}{1 - 4} = \frac{30}{-3} = -10$$

$$q = -10p + b$$

$$50 = -10(1) + b$$

$$b = 60$$

$$q = -10p + 60$$

Supply

$$q = mp + b$$

$$m = \frac{50 - 20}{5 - 2} = \frac{30}{3} = 10$$

$$q = 10p + b$$

$$20 = 10(2) + b$$

$$b = 0$$

$$q = 10p$$

b) At what price should the DVD rentals be marked so that there is neither a surplus nor a shortage of rental DVDs?

$$-10p + 60 = 10p$$

$$60 = 20p$$

$$p = \frac{60}{20} = 3$$

3) The city of Metropolis is growing exponentially, with a doubling time of 29 years.

- a) Obtain an exponential growth model  $P(t) = P_0 e^{kt}$  for Metropolis. (Round coefficients to four decimal places.)

$$k = \frac{\ln(2)}{29} = 0.0239$$

$$P(t) = P_0 e^{0.0239t}$$

- b) If the population of Metropolis was 5.2 million in 2004, how many people does your model predict will live there in 2014?

$$P(t) = 5.2 e^{0.0239t}$$

$$P(10) = 5.2 e^{0.0239(10)} = 6.6$$

About 6.6 million people

4) The demand function for a specific product is given by  $q = 55 - \frac{1}{4}p$  units, where  $p$  is the price per unit.

a) Find the revenue function  $R(p)$ .

$$R(p) = pq = p\left(55 - \frac{1}{4}p\right) = 55p - \frac{1}{4}p^2$$

b) Find the price that maximizes the revenue.

$$-\frac{b}{2a} = \frac{-55}{2\left(-\frac{1}{4}\right)} = \# 110$$

c) Find the maximum revenue.

$$R(110) = 55(110) - \frac{1}{4}(110)^2 = \# 3,025$$

d) How many units must be produced to maximize the revenue?

$$q = 55 - \frac{1}{4}(110) = 27.5 \quad \text{or } 28 \text{ units}$$

- 5) A certain bicycle factory can produce 10 bicycles in a day at a total cost of \$5,500 and it can produce 30 bicycles a day at a total cost of \$6,500. What are the company's daily fixed costs, and what is the marginal cost per bicycle?

$$C(x) = mx + b$$

$$m = \frac{6500 - 5500}{30 - 10} = \frac{1000}{20} = 50$$

$$C(x) = 50x + b$$

$$5500 = 50(10) + b$$

$$5000 = b$$

$$C(x) = 50x + 5000$$

fixed costs are \$5000

marginal costs are \$50 per bicycle